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Type III Compensator Design for Power Converters

A detailed analysis of the type III compensator derives the appropriate equations and guarantees the targeted bandwidth and phase margin, as well as an unconditionally stable control loop.

Type II compensators are widely used in the control loops for power converters. However, there are cases where the phase lag of a power converter can approach 180 degrees, while the maximal phase from a type II compensator at any frequencies is at most zero degree. Thus in these cases, the type II compensator cannot provide enough phase margin to keep the loop stable, and this is where a type III compensator is needed. A type III compensator can have a phase plot going above zero degree at some frequencies, and therefore it can provide the required phase boost to maintain a reasonable phase margin.

Although the concept of the type III compensator has been around for years, an in-depth analysis on the compensator is not easy to find. There are some design procedures described in the literature [1,2,3,4]. However, these procedures are usually empirically derived, and the derivation processes are not provided, which make it difficult to follow and evaluate these procedures.

An analog implementation of type III compensators is shown in *Fig. 1*, where six passive circuit components are needed. The transfer function of the Type III compensator in *Fig. 1.* is given by:

$$C(s) = \frac{v_o}{v_i} = -\frac{(sC_2R_2+1)[sC_3(R_1+R_3)+1]}{R_1(C_1+C_2)s(sC_{12}R_2+1)(sC_3R_3+1)} \quad (1)$$

where C_{12} is the parallel combination of C_1 and C_2,

$$C_{12} = \frac{C_1C_2}{C_1+C_2} \quad (2)$$

The Type III compensator has three poles (one at the origin) and two zeros. In practice, it is usually arranged to have two coincident zeros and two coincident poles, and the loop crossover frequency is placed somewhere between the zeros and poles. For this kind of design, the transfer function in Equation (1) can be rewritten as:

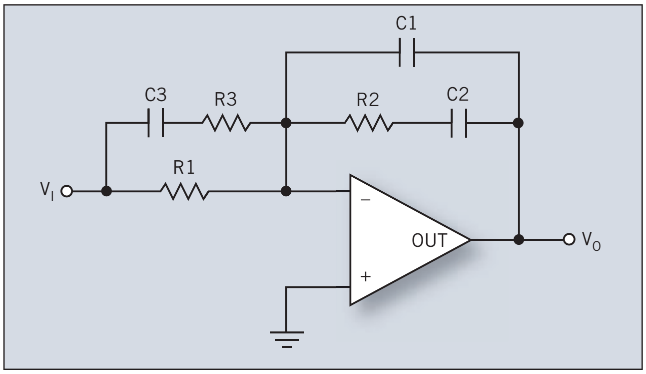

Fig. 1. A type III error amplifier configuration employs six passive circuit components and has three poles (one at the origin) and two zeros.

$$C(s) = \frac{K\left(\frac{s}{\omega_Z}+1\right)^2}{s\left(\frac{s}{\omega_P}+1\right)^2} \quad (3)$$

where the zero's and pole's frequencies are given by:

$$\omega_z = \frac{1}{C_2R_2} = \frac{1}{C_3(R_1+R_3)}$$
$$\omega_p = \frac{1}{C_{12}R_2} = \frac{1}{C_3R_3} \quad (4)$$

and the constant gain K is given by:

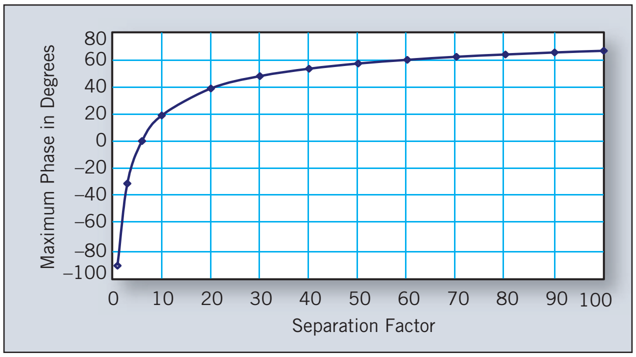

Fig. 2. The plot of maximum phase vs. separation factor shows that the phase increases quickly when the separation factor k is small, and becomes flatter as k gets higher.

$$K = \frac{1}{R_1(C_1 + C_2)} \quad (5)$$

The amplitude of the transfer function in Equation (3) at a given frequency ω can be calculated as:

$$|C(j\omega)| = \frac{K}{\omega}\frac{\left|\left(1+j\frac{\omega}{\omega_z}\right)^2\right|}{\left|\left(1+j\frac{\omega}{\omega_p}\right)^2\right|} = \frac{K}{\omega}\frac{\left|\left(1+j\frac{\omega}{\omega_z}\right)\right|^2}{\left|\left(1+j\frac{\omega}{\omega_p}\right)\right|^2} = \frac{K}{\omega}\frac{1+\left(\frac{\omega}{\omega_z}\right)^2}{1+\left(\frac{\omega}{\omega_p}\right)^2} \quad (6)$$

The phase of the transfer function in Equation (3) at a given frequency ω can be calculated as:

$$\begin{aligned}\phi[C(j\omega)] &= \phi\left(\frac{K}{j\omega}\right) + \phi\left(1+j\frac{\omega}{\omega_z}\right)^2 - \phi\left(1+j\frac{\omega}{\omega_p}\right)^2 \\ &= -\frac{\pi}{2} + 2\phi\left(1+j\frac{\omega}{\omega_z}\right) - 2\phi\left(1+j\frac{\omega}{\omega_p}\right)\end{aligned} \quad (7)$$

As can be seen, the phase of C(jω) has two parts: a constant part of -π/2 due to the pole at the origin, and a variable part as a function of frequency ω given by:

$$\begin{aligned}\phi_v(\omega) &= 2\phi\left(1+j\frac{\omega}{\omega_z}\right) - 2\phi\left(1+j\frac{\omega}{\omega_p}\right) \\ &= 2\left(\tan^{-1}\frac{\omega}{\omega_z} - \tan^{-1}\frac{\omega}{\omega_p}\right)\end{aligned} \quad (8)$$

Equation (8) can be converted to:

$$\phi_v(\omega) = 2\tan^{-1}\frac{\omega(\omega_p - \omega_z)}{\omega^2 + \omega_z\omega_p} \quad (9)$$

Equation (9) has a useful feature in that the function reaches its maximum value somewhere between ω_z and ω_p. This can be shown as follows. Note that the inverse tangent function is monotonically increasing. Therefore, to find the maximum value of $\phi_v(\omega)$, we can first search for the maximum of the following function:

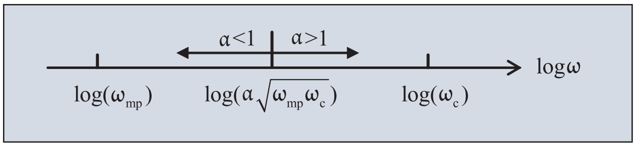

Fig. 3. To find the maximum phase frequency placement for a type III compensator we can use α to adjust the location of the maximum phase point.

$$F(\omega) = \frac{\omega(\omega_p - \omega_z)}{\omega^2 + \omega_p\omega_z} \quad (10)$$

The maximum value of F(ω) can be found through its derivative, which is calculated as:

$$\frac{dF}{d\omega} = \frac{(\omega_p - \omega_z)(\omega_z\omega_z - \omega^2)}{(\omega^2 + \omega_z\omega_z)^2} \quad (11)$$

Based on Equation (11), you find that F(ω), and hence $\phi_v(\omega)$, reaches their maximum values at the frequency defined by:

$$\omega_m = \sqrt{\omega_p\omega_z} \quad (12)$$

Equation (12) says that the maximum phase of $\phi_v(\omega)$ occurs at the geometric mean of ω_z and ω_p. Here, we call ωm the maximum phase frequency of a type III compensator. By substituting Equation (12) into (9), you get the maximum phase of $\phi_v(\omega)$ as:

$$\phi_v(\omega_m) = 2\tan^{-1}\frac{\sqrt{\omega_p\omega_z}(\omega_p - \omega_z)}{2\omega_p\omega_z} = 2\tan^{-1}\frac{(\omega_p - \omega_z)}{2\sqrt{\omega_p\omega_z}} \quad (13)$$

Define the ratio of the pole's frequency to the zero's frequency as:

$$k = \frac{\omega_p}{\omega_z} \quad (14)$$

From Equation (12) and (14), k can also be defined as:

$$\sqrt{k} = \frac{\omega_m}{\omega_z} = \frac{\omega_p}{\omega_m} \quad (15)$$

Then the maximum phase of $\phi_v(\omega)$ can be written as:

$$\phi_v(\omega_m) = 2\tan^{-1}\frac{k-1}{2\sqrt{k}} \quad (16)$$

And the maximum phase of the type III compensator is given by:

$$\phi[C(j\omega_m)] = -\frac{\pi}{2} + 2\tan^{-1}\frac{(k-1)}{2\sqrt{k}} \quad (17)$$

Note that k is a measure on the distance between the

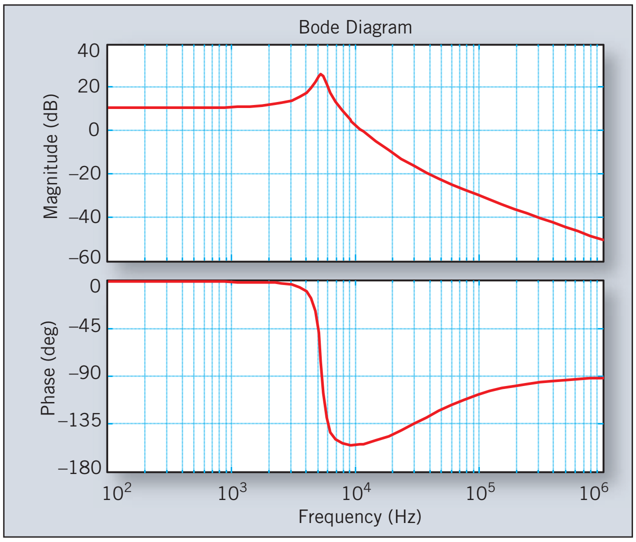

Fig. 4. Control plant's Bode plot for a synchronous buck converter, including the PWM modulator.

zero and pole, and hence we call it separation factor. With Equation (17), you can calculate the maximum phase boost of the type III compensator for a given separation factor, or vise versa.

The maximum value of an inverse tangent function is 90°. Base on Equation (17), the maximum phase boost from a type III compensator is 90°. *Fig. 2* shows the maximum phase of a type III compensator vs. the separation factor. As can be seen, the phase increases quickly when the separation factor k is small, and it becomes more and more flat as the factor goes high. Thus, in the low range of k, it is more effective to adjust the phase boost from a type III compensator by changing the separation factor. It is worth to note that there is a range for the separation factor where the phase of the type III compensator is negative. Since the main purpose of using a type III compensator is to boost the control loopís phase, it is useful for the designer to know the value of the separation factor at which the phase is zero. From Equation (17) you can find that the phase is zero when the inverse tangent function meets:

$$\tan^{-1}\frac{k-1}{2\sqrt{k}} = \frac{\pi}{4} \quad (18)$$

From Equation (18) we can see that k needs to meet:

$$k - 2\sqrt{k} - 1 = 0 \quad (19)$$

By solving Equation (19), we get the value of k that gives zero phase boost for a type III compensator:

$$k = (1+\sqrt{2})^2 = 5.827 \quad (20)$$

As a rule of thump, the separation factor of a type III compensator should be larger than 6 in order to provide a positive phase boost to the loop.

DESIGN PROCEDURES

Procedure I

With Procedure I, you place the loop crossover frequency (ω_c) at ω_m, and in this way you can reach the maximal loop phase margin with a given separation factor. In the following, a design procedure is derived to achieve this design goal.

Let the control plant's gain and phase at ω_c be Gp and ϕ_p, and the desired phase margin be ϕ_m. To meet the phase margin requirement, we should have:

$$-\frac{\pi}{2} + \phi_v(\omega_c) + \phi_p + \pi = \phi_m \quad (21)$$

Thus, we can get the phase $\phi_v(\omega_c)$ as follows:

$$\phi_v(\omega_c) = \phi_m - \phi_p - \frac{\pi}{2} \quad (22)$$

Since we have chosen $\omega_c = \omega_m$, thus from Equation (16) and (22) we get:

$$\tan^{-1}\frac{k-1}{2\sqrt{k}} = \frac{\phi_m - \phi_p}{2} - \frac{\pi}{4} \quad (23)$$

Or equivalently, we have:

$$\frac{k-1}{2\sqrt{k}} = b \quad (24)$$

where b is defined by:

$$b = \tan\left(\frac{\phi_m - \phi_p}{2} - \frac{\pi}{4}\right) \quad (25)$$

Define:

$$x = \sqrt{k} \quad (26)$$

Then, from Equation (24) we get the following quadratic equation in terms of x:

$$x^2 - 2bx - 1 = 0 \quad (27)$$

The solutions to Equation (27) are given by:

$$x = b \pm \sqrt{b^2+1} \quad (28)$$

From Equation (26) you can see that x is positive, therefore the solution we need is given by:

$$\sqrt{k} = x = b + \sqrt{b^2+1} \quad (29)$$

Given ωm and k, we can get the zero ω_z and pole ω_p based on Equation (15):

$$\omega_z = \frac{\omega_m}{\sqrt{k}}, \omega_p = \sqrt{k}\omega_m \quad (30)$$

From Equation (6), we can get the compensator's gain at the crossover frequency, $\omega_c = \omega_m$:

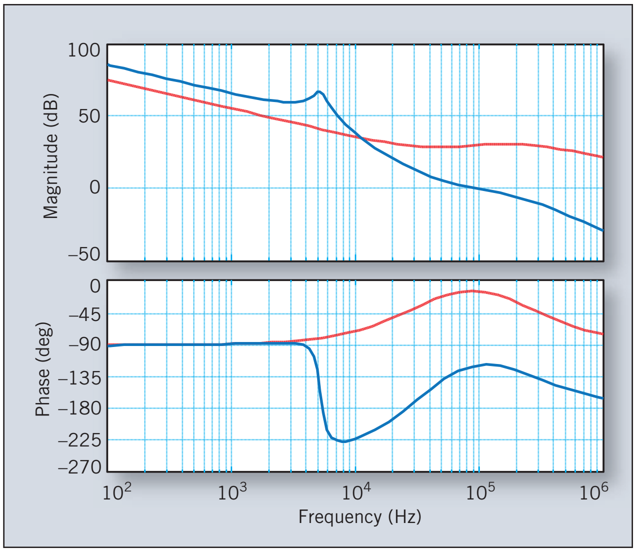

Fig. 5. Compensator (red) and loop (blue) Bode plots from Procedure 1 is conditionally stable.

$$\left|C(j\omega_m)\right| = \frac{K}{\omega_m}\frac{1+\left(\frac{\omega_m}{\omega_z}\right)}{1+\left(\frac{\omega_m}{\omega_p}\right)^2} = \frac{K}{\omega_m}\left(\frac{1+k}{1+\frac{1}{k}}\right) = \frac{Kk}{\omega_m} \quad (31)$$

At the crossover frequency ω_c, the loop gain is equal to 1, that is:

$$\frac{Kk}{\omega_m}G_p = 1 \quad (32)$$

From Equation (32) we get:

$$K = \frac{\omega_m}{G_p k} \quad (33)$$

As shown in [2] and [3], the design of a type III compensator usually starts with choosing a value for R_1.With R_1 chosen, the rest components values can be calculated as follows.

From Equation (5), we get:

$$C_1 + C_2 = \frac{1}{R_1 K} \quad (34)$$

Equation (4) is equivalent to the following four equations:

$$C_2 R_2 = \frac{1}{\omega_z} \quad (35)$$

$$C_3(R_1 + R_3) = \frac{1}{\omega_z} \quad (36)$$

$$C_3 R_3 = \frac{1}{\omega_p} \quad (37)$$

$$C_{12} R_2 = \frac{1}{\omega_p} \quad (38)$$

Equations (34) to (38) are the five equations that we need to determine the componentsí values R_2, R_3, C_1, C_2 and C_3.

By subtracting Equation (37) from Equation (36), we get the value of C_3:

$$C_3 = \frac{1}{R_1}\left(\frac{1}{\omega_z} - \frac{1}{\omega_p}\right) \quad (39)$$

From Equation (35) we have:

$$C_1 C_2 R_2 = \frac{C_1}{\omega_z} \quad (40)$$

Equation (38) can be rewritten as:

$$\frac{C_1 C_2 R_2}{C_1 + C_2} = \frac{1}{\omega_p} \quad (41)$$

By Substituting Equation (34) and Equation (40) into Equation (41), we get:

$$\frac{C_1 R_1 K}{\omega_z} = \frac{1}{\omega_p} \quad (42)$$

From Equation (42), we get the solution for C_1:

$$C_1 = \frac{\omega_z}{\omega_p R_1 K} \quad (43)$$

From Equation (34), we get the solution for C_2:

$$C_2 = \frac{1}{R_1 K} - C_1 = \frac{1}{R_1 K}\left(1 - \frac{\omega_z}{\omega_p}\right) \quad (44)$$

Now that we have determined the values for all of the capacitors. The resistor values can be obtained from Equation (37) and Equation (38):

$$R_2 = \frac{1}{C_{12}\omega_p} \quad (45)$$

$$R_3 = \frac{1}{C_3\omega_z} - R_1 \quad (46)$$

Summarizing Procedure 1 for the Type III compensator, we find that:

Given desired crossover frequency ω_c and phase margin ϕ_m, and the control plant's gain and phase at ω_c as Gp and ϕ_p.

1. Calculate the tangent value b using:

$$b = \tan\left(\frac{\phi_m - \phi_p}{2} - \frac{\pi}{4}\right) \quad (47)$$

2. Calculate the zero and pole separation factor k:

$$\sqrt{k} = b + \sqrt{b^2 + 1} \quad (48)$$

3. Calculate the zero's and pole's frequency:

$$\omega_z = \frac{\omega_c}{\sqrt{k}} \quad \omega_p = \sqrt{k}\omega_c \quad (49)$$

4. Calculate the compensator's gain K:

$$K = \frac{\omega_c}{G_p k} \quad (50)$$

5. Select a resistor value for R_1.
6. Calculate C_3:

$$C_3 = \frac{1}{R_1}\left(\frac{1}{\omega_z} - \frac{1}{\omega_p}\right) \quad (51)$$

7. Calculate C_1:

$$C_1 = \frac{\omega_z}{\omega_p R_1 K} \quad (52)$$

8. Calculate C_2:

$$C_2 = \frac{1}{R_1 K} - C_1 \quad (53)$$

9. Calculate R_2:

$$R_2 = \frac{(C_1 + C_2)}{C_1 C_2 \omega_p} \quad (54)$$

10. Calculate R_3:

$$R_3 = \frac{1}{C_3 \omega_z} - R_1 \quad (55)$$

11. Verify the calculated compensator's values and frequency response.
12. Check the closed loop's frequency response.

Procedure II

A type III compensator is usually used for the control plant that has a big phase lag around the loop crossover frequency range. For this type of plants, the control loop may end up to be conditionally stable, which is not desired in some applications. In the following, a design procedure is derived which takes unconditional stability into consideration.

A conditionally stable loop is the one whose phase plot goes more negative than -180°, but comes back above -180° again before the crossover frequency. This occurs usually around the frequency where the plant has the maximum phase lag. We name this frequency as ω_{mp}.

To make the loop unconditionally stable, some phase boost is needed at ω_{mp}. On the other hand, to make the loop stable, a certain amount of phase boost is also needed at the crossover frequency ω_c. To meet these requirements, one can place the maximum phase frequency ω_m (defined by Equation 12) somewhere between ω_{mp} and ω_c. The placement of ω_m can be described by the following:

$$\omega_m = \alpha\sqrt{\omega_{mp}\omega_c} \quad (56)$$

where ∝is a number to be determined.

At the logarithmically scaled frequency axis, the geometric mean of ω_{mp} and ω_c has an equal distance to ω_{mp} and ω_c, as shown in Fig. 3. You can see that if ∝ < 1, then ωm is closer to ω_{mp} than to ω_c. On the other hand, if ∝> 1, then ωm is closer to ω_c than to ω_{mp}. Therefore, you can use ∝ to adjust the location of the maximum phase point of a type III compensator, and by taking a trial-and-error search on ∝, you can eventually find the proper location of ωm that meets both of the phase margin and unconditional stability requirements.

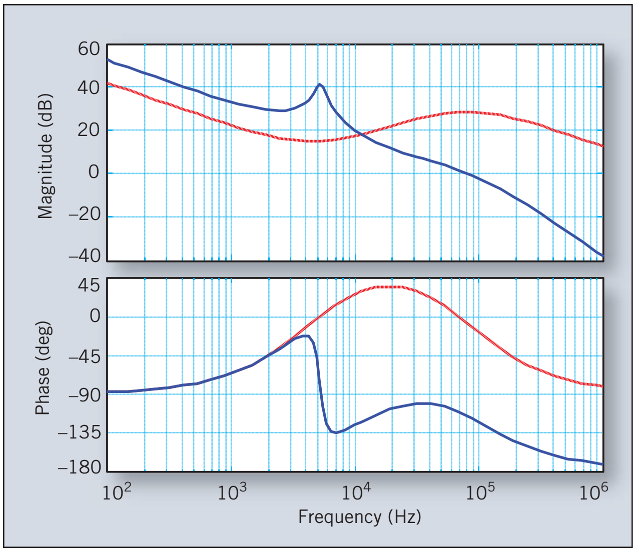

Fig. 6. Compensator (red) and loop (blue) Bode plots from Procedure 2 is unconditionally stable.

As soon as ω_m has been selected, the type III compensator can be calculated as follows. Given the control plant's gain and phase at ω_c: Gp and ϕ_p, and the desired phase margin ϕ_m. Also given the frequency ω_{mp} at which the plant has the maximum phase lag. At the crossover frequency, based on Equation (9) we have:

$$\phi_v(\omega_c) = 2\tan^{-1}\frac{\omega_c(\omega_p - \omega_z)}{\omega_c^2 + \omega_z\omega_p} \quad (57)$$

To meet the phase margin requirement, we need to satisfy Equation (22), which in turn leads to:

$$2\tan^{-1}\frac{\omega_c(\omega_p - \omega_z)}{\omega_c^2 + \omega_z\omega_p} = \phi_m - \phi_p - \frac{\pi}{2} \quad (58)$$

Or, equivalently:

$$\frac{\omega_c(\omega_p - \omega_z)}{\omega_c^2 + \omega_p\omega_z} = \tan\left(\frac{\phi_m - \phi_p}{2} - \frac{\pi}{4}\right) \quad (59)$$

Based on Equation (12) and Equation (59) we can get the following two equations about ω_p and ω_z:

$$\omega_z\omega_p = \omega_m^2 \quad (60)$$

$$\omega_p - \omega_z = \omega_d \quad (61)$$

where ω_d is defined by:

$$\omega_d = \tan\left(\frac{\phi_m - \phi_p}{2} - \frac{\pi}{4}\right)\left(\omega_c + \omega_{mp}\right) \quad (62)$$

Note that ω_d is known with the given parameters ϕ_m, ϕ_p, and ω_p, and the selected frequency ω_c.

We can solve Equation (60) and (61) and get the compensator's zero and pole frequencies:

$$\omega_z = 0.5\left(\sqrt{\omega_d^2 + 4\omega_m^2} - \omega_d\right) \quad (63)$$

$$\omega_p = 0.5\left(\sqrt{\omega_d^2 + 4\omega_m^2} + \omega_d\right) \quad (64)$$

The separation factor can be calculated as:

$$k = \frac{\sqrt{\omega_d^2 + 4\omega_m^2} + \omega_d}{\sqrt{\omega_d^2 + 4\omega_m^2} - \omega_d} \quad (65)$$

With ω_z, ω_p and k determined based on the above equations, the compensator's components can be determined in the same way as in Procedure I. The design procedure that accounts for unconditional stability is summarized below.

Given desired crossover frequency ω_c and phase margin ϕ_m, and the control plant's gain and phase at ω_c as G_p and ω_p. Also given the frequency ω_{mp} at which the plant has the maximum phase lag.

Based on Equation (56) determine the compensator's maximum phase frequency by choosing a value for ∝. Usually you can start with ∝ =1, and adjust it based on the loop's Bode plot resulted from the design procedure.

1. Calculate the difference between the zero's frequency and pole's frequency using Equation (62).

2. Calculate the zero's frequency ω_z and pole's frequency ω_p using Equation (63) and Equation (64).

3. Calculate the separation factor k using Equation (65).

4. From Equation (6) we have

$$|C(j\omega_c)| = \frac{K}{\omega_c}\frac{1 + \left(\frac{\omega_c}{\omega_z}\right)^2}{1 + \left(\frac{\omega_c}{\omega_p}\right)^2} \quad (66)$$

At the crossover frequency:

$$|C(j\omega_c)|G_p = 1 \quad (67)$$

Thus, we can calculate the compensator's constant gain K as:

$$K = \frac{\omega_c\left[1 + \left(\frac{\omega_c}{\omega_p}\right)^2\right]}{G_p\left[1 + \left(\frac{\omega_c}{\omega_z}\right)^2\right]} \quad (68)$$

With K determined, one can follow the steps 5 through 12 in Procedure I to finish the design.

The synchronous dc-to-dc converter shown in *Fig. 4* [2] will be used an example for applying the design procedures.In [2], the target bandwidth is set to 90kHz, and the phase margin is required to be larger than 45°. Here, the same bandwidth is targeted, and the phase margin is targeted at 60°. From *Fig. 4*, one can find that at 90kHz, the plant's gain is -29.14dB or 0.0349, and the phase is -109.1°.

First, Procedure I will be used to calculate the compensator. With this approach, we place the compensator's maximum phase boost frequency at the target crossover frequency, that is, $\omega_m=2\times\pi\times90\times10^3$rad/s. By choosing R_1 = 2kΩ and following the procedure, we get the following component values:

R_1 = 2kΩ
R_2 = 34.7kΩ
R_3 = 571Ω,
C_1 = 31pF
C_2 = 108pF
C_3 = 1.5nF

F ig. 5. shows the compensator's and the resulting loop's Bode plots. The separation factor is 4.5 and the phase plot of the compensator is under zero degree as shown in *Fig. 5*. The loop bandwidth is 90kHz and phase margin is 60°. However, the phase plot goes more negative than -180°, thus making the loop only conditionally stable.

Utilizing Procedure II, the first step is to locate the compensator's maximum phase boost frequency. From *Fig. 4*, the maximum phase lag frequency is about 9kHz. Thus, ω_m should be somewhere between 9kHz and 90kHz. Based on Equation (56), it was found that with ∝=0.7 we can get a good unconditionally stable control loop. With this value of ∝, the following component values result:.

R_1 = 10kΩ
R_2 = 30.4kΩ
R_3 = 568Ω,
C_1 = 66pF
C_2 = 1.2nF
C_3 = 3.4nF

Fig. 6. shows the resulting loop's Bode plots. As you can see, the loop in *Fig. 6* is unconditionally stable, as opposed to that in *Fig. 5*.

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